Relevant reading

Must-read articles


Section 1

A better QTM: Quantity theory of credit
Recall Fisher’s original formulation of the equation of exchange:

\[ M \cdot V_{\text{transactions}} = \sum_i (p_i \cdot q_i) \]

where \( M \) is the average total amount of money in circulation over a period, \( V_{\text{transactions}} \) is the transactions velocity of money, and \( p \) and \( q \) are vectors of prices and quantities of all transactions.

i.e. the above states a truism that the total value of transactions during any time period must be the same as the amount of money used to pay for these transactions.
At the time, the equation of Fisher had an important drawback for practical application.

While $M$ and $p$ could be readily identified, $V$ was hard to measure and thus had to be the residual.

One would thus still require data on transactions – $q$.

This data, however, did not exist in official publication\(^1\).

Lacking this data, and given the growth of availability of national income accounts, the right hand side of the Fisher equation was replaced with total expenditures ($P \cdot Y$) as calculated in GNP.

\(^1\)In modern days, central banks in developed economies could easily publish this data in real time since they act as clearing houses for bank transactions that account for 95%+ of all transaction values.
Indeed, as Friedman later noted:

“Fisher, in his original version, used $T$ to refer to all transactions – purchases of final goods and services [...], intermediate transactions [...], and capital transactions (the purchase of a house or a share of stock). In current usage, the item has come to be interpreted as referring to purchases of final goods and services only, and the notation has been changed accordingly, $T$ being replaced by $y$, as corresponding to real income.”


The justification for this replacement, however, was not all too accurate.
We know that in the modern world, asset transactions constitute an important potential use of the money stock, $M$, but they are not included in the GDP statistics, since capital gains on assets are not included in the definition of income under GDP calculation.

Therefore the usual $MV = PY$ equation will not be reliable when the value of asset (non-GDP) transactions is significant. The rise in the volume of such non-GDP transaction compared to GDP transactions will thus be interpreted by the above equation as an appearance of a fall in money velocity, $V$.

The solution to this drawback, due to Richard Werner, is to disaggregate the general equation of exchange for all transactions into two flows – those of money used for real (GDP) transactions and those of money used for financial (non-GDP) transactions:

$$MV = M_R V_R + M_F V_F$$

$$PQ = P_R Q_R + P_F Q_F$$

$$\equiv P_R Y$$
From the above construction, it must hold that:

\[ M_R V_R = P_R Y \]
\[ M_F V_F = P_F Q_F \]

Taking net changes we get:

\[ \Delta (M_R V_R) = \Delta (P_R Y) \]
\[ \Delta (M_F V_F) = \Delta (P_F Q_F) \]

i.e.

- the rise (fall) in the amount of money used for GDP transactions is equal to the rise (fall) in nominal GDP, and
- the rise (fall) in the amount of money used for non-GDP transactions is equal to the change in the value of non-GDP transactions.\(^2\)

\(^2\)Which lends itself nicely to the interpretation that an asset bubble can be caused if more money is created and injected into particular asset markets.
Werner then goes on to argue that, since most money in modern economies is created by the banking system via the creation of new credit, we should actually replace the money stock, $M$, by the amount of credit:

\[ C_R V_R = P_R Y \]
\[ C_F V_F = P_F Q_F \]

And once again taking net changes we get:

\[ \Delta (C_R V_R) = \Delta (P_R Y) \]
\[ \Delta (C_F V_F) = \Delta (P_F Q_F) \]

i.e.

- real circulation credit determines nominal GDP growth, and
- financial circulation credit determines asset prices (and leads to asset cycles and banking crises).
Credit ($\Delta \left( C_R V_R \right)$) and nominal GDP ($\Delta \left( P_R Y \right)$)
Credit ($\Delta \left( C_F V_F \right)$) and asset prices
Thus, bank credit creation to a very significant effect determines economic growth.

The effect of bank credit allocation depends on where the use money is put:

- **Productive** credit creation (i.e. investment credit)
  - for new goods, services and productivity growth – that results in non-inflationary growth (possibly even at full employment).

- **Unproductive** credit creation
  - for consumption – that results inflationary growth (or just inflation), and
  - for financial (non-GDP) transactions – that results in asset price inflation, and potentially to asset price bubbles.
Section 2

Money neutrality, inflation expectations and real interest rates
In the previous lecture we saw that in classical theory money was neutral, not having any effects on real variables such as consumption or output.

This implies that in static equilibrium all real variables in the economy are independent of the quantity of nominal money and the price level is proportional to the quantity of money.

Now we turn to considering whether the welfare effects of money and inflation.

The classical model assumes that prices are perfectly flexible and the economy is always in equilibrium with full employment.

There also exists the notion of superneutrality, i.e. that the rate of growth of the quantity of money is also neutral in the sense that it only affects the rate of growth of the price level (inflation) but does not affect real variables.
Inflation in such a setup would indeed be a monetary phenomenon with no real effects.

We will find that the above implication of superneutrality is not correct, since expectations of the future growth rate of the quantity of money create expectations of inflation, and the latter are not neutral.

This last effect is captured by what is known as the *Fisher equation*. 
Let us at this point start distinguishing between the nominal and real interest rate – the interest rate that is adjusted by subtracting expected changes in the price level (inflation) so that it more accurately reflects rate at which goods today can be traded for goods in the future, which is linked to the true cost of borrowing.

This real interest rate is more precisely referred to as the \textit{ex ante} real interest rate, since it is adjusted for \textit{expected} changes in the price level. This is the rate that is most important to economic decisions.

The interest rate that is adjusted for \textit{actual} realized changes in the price level is called the \textit{ex post} real interest rate, and it reflects the real gains realized by a lender after considering the effects of inflation.

The real interest rate is more accurately defined from the Fisher equation, due to Irving Fisher:

\[ i = r + \pi^e \]

i.e. the nominal interest rate equals the real interest rate plus the expected rate of inflation.
Let us see how the above relation comes about:

- Assume someone buys a $1 bond in period $t$ while the interest rate is $i_t$. If redeemed in period $t+1$, the buyer will receive $(1 + i_t)$ dollars. But if the price level has changed between periods $t$ and $t+1$, then the ex-post real value, adjusted for the effects of inflation, of the proceeds from the bond therefore is equal:

$$\left(1 + r_{t+1}\right) = \frac{1 + i_t}{1 + \pi_{t+1}}$$

- which, rearranging for the nominal rate gives:

$$1 + i_t = \left(1 + r_{t+1}\right)\left(1 + \pi_{t+1}\right)$$

$$= 1 + r_{t+1} + \pi_{t+1} + r_{t+1}\pi_{t+1}$$

- Subtracting 1 from both sides produces:

$$i_t = r_{t+1} + \pi_{t+1} + r_{t+1}\pi_{t+1} \approx \text{very small}$$

$$\approx r_{t+1} + \pi_{t+1}$$

- and finally replacing actual inflation by expected inflation yields the above Fisher equation, valid as an approximation for small changes in $r_t$ and $\pi_t$. 
Section 3

Superneutrality
Superneutrality may be defined as the proposition that, where inflation is fully anticipated – i.e. $\pi_{t+1} = \pi_{t+1}^e$ –, the real interest rate, $r_t$, is independent of the fully anticipated inflation rate, $\pi_{t+1}$.

The real interest rate measures the relative price of goods in the future as against goods today, which is most relevant to inter-temporal decisions.

If it were to change in response to a change in the inflation rate, then inflation would not be neutral in its effects on such decisions.

However, if the real interest rate does not change in response to a change in the inflation rate, then from the Fisher equation, it follows that the nominal rate, $i_t$, must adjust one-for-one to changes in the inflation rate.
Assuming a closed economy in a standard macro model for now, we can write the output equation as:

\[ Y = C \left( Y^+ - T, \bar{r} \right) + I \left( \bar{Y}, \bar{r} \right) + G \]

where:
- \( Y \) – is output/income,
- \( C \) – is consumption,
- \( T \) – are taxes on income (and hence \( Y - T \) is disposable income),
- \( I \) – is investment, and
- \( G \) – is government spending.

The classical model adds further structure by assuming a negative relationship between the real interest rate and output:

\[ r = f \left( \bar{Y} \right) \]
Plugging this into the \textit{ex-post} Fisher equation produces the IS curve:

\[ i = f \left( \bar{Y} \right) + \pi \]

that shows the different combinations of output and nominal interest rate that clear the goods market.

Therefore, in the standard IS–LM model with the nominal interest rate on the vertical axis, an increase in the inflation rate will shift the IS curve upwards but leave the position of the LM curve unchanged.

There must therefore be a one-off increase in the price level to restore equilibrium in the money market at the given level of equilibrium output.
The increase in the nominal interest rate reduces the demand for real money balances and the jump in the price level reduces the supply.

In this standard model, changes in the fully anticipated inflation rate leave the real interest rate unchanged but affect not only the nominal interest rate but also the stock of real money balances, which leads to the welfare costs of fully anticipated inflation.
Reintroducing wealth effects in the consumption function prompts a reformulation of the IS curve equations as:

\[ Y = C \left( Y - T, r, \frac{M}{P} \right) + I \left( Y, r \right) + G \]

\[ r = f \left( \frac{M}{P} \right) \]

where \( \frac{M}{P} \) – denotes real money balances.

Thus for any given output level, a fall in real money balances necessitates a fall in real interest rates in order to offset the decrease in aggregate demand.
In the goods market equilibrium there is then a positive relationship between $M/P$ and $r$.

Following the increase in inflation, caused by the positive growth rate of nominal money balances, the IS curve shifts out as but is partially offset because of the fall in wealth (fall in $M/P$) which reduces consumption.

The nominal interest rate only increases less than before, and, since inflation is the same, the real interest rate falls.
Section 4

Welfare costs of inflation
From our previous study of money demand we know that it negatively relates to the nominal interest rate.

From the Fisher equation, assuming perfect capital markets, we know that the nominal interest rate increases to match increases in the expected rate of inflation.

Hence, expected inflation decreases holdings of real money balances, the demand for real money balances will have a downward sloping curve when plotted against inflation, which itself is part of the opportunity cost of holding these balances – the nominal interest rate.

In the above analysis we have assumed that real money balances figure directly in the agent’s utility function – such that smaller holdings imply less utility.

If money can be created at no cost and does not pay any interest, the area under the demand curve for real money balances can be used as a measure of the loss of consumer associated with an increase in interest rates.
For an economy in which money is neutral (so that it does not change output) and assuming that money balances pay no interest, the consumers’ surplus enjoyed by individuals from holding money balances – i.e. the excess of the utility they derive over the opportunity cost they incur – is thus measured by the area $A$ below:

The welfare cost of holding smaller money balances at the nominal interest rate $R_0$ is thus measured by the sum of areas $B + C$, since this total area measures the consumer surplus lost as a result of a positive nominal interest rate.
In this simplistic view of monetary interactions, a term *seigniorage* is used to denote the potential transfer of resources from the private sector to the government that occurs when the government makes purchases with freshly created money at no cost of such creation – the government’s income stream from such seigniorage is denoted by the area B above.

Area C is thus the excess burden or deadweight loss associated with this seigniorage tax.
Holding of real money balances in excess of \( \left( \frac{M}{P} \right)_{0} \) still yield positive utility and the production of additional money is costless. Therefore, economic benefit could be derived from expanding the quantity of real money balances held from \( \left( \frac{M}{P} \right)_{0} \) up to the point L.

However, because at the nominal rate of \( R_{0} \) the desired holding of money balances is \( \left( \frac{M}{P} \right)_{0} \), and thus any additional nominal money will only lead to an increase in the price level to restore the desired quantity of real balances.
Consider a fully anticipated inflation at a rate of $\pi_1$, such that the nominal interest rate in the above diagram is now $R_1 = r + \pi_1$ and the demand for real money balances is $\left(\frac{M}{P}\right)_1$.

- The opportunity cost of holding money is area $A$.
- The revenue the government now derives from money issue is $B$ (inflation tax) + $D$ (seigniorage = real interest rate times real money balances).
Welfare cost is $C + E$ – the reduction in the quantity of real money balances demanded multiplied by the nominal interest rate averaged over that range.

The welfare cost of fully anticipated inflation arises because inflation raises the nominal interest rate, reducing the demand for real money balances (due to increased private cost of holding them) and therefore reducing the utility obtained from the use of money.
Inflation reduces demand, has welfare costs, and also raises revenue for the government – so it is in a way similar to taxation, though its cost may be lower than those of explicit taxation.

Net increase in government revenue from inflation is the inflation tax, area $B$, less the reduced yield from seigniorage, area $E$. 
We will return to a more detailed critical look at the IS-LM model later, but for now we note:

1. It assumes that monetary policy is performed through control of the money supply, whereas in reality it is done via the control of short-term interest rates.

2. It does not effectively distinguish between the real interest rate, which determines investment, and the nominal interest rates, which determines the demand for money and bonds. From the Fisher equation, the difference between these rates is the expected inflation rate, the determination of which requires a theory of expectations in addition to a macroeconomic model for determining the inflation rate, which is a dynamic variable.

3. It does not recognize more than two distinct financial assets, money and bonds, whereas more than two such assets (such as credit and other financial instruments) could enrich the analysis and explain additional aspects of the macroeconomy.

4. It is focused on the short run in which the capital stock, technology and labor force are held fixed. For certain types of analysis, such as for explaining business cycles and growth, one needs to allow these to vary.
Section 5

Inflation, hyperinflation and government deficits
A simple model of hyperinflations

A typical definition of hyperinflation is a very rapid, and in particular very rapidly accelerating, inflation that leads to an explosion of the price level and ultimately to a collapse of the monetary system.

Cagan (1956), in his most celebrated paper, offered a model of money demand that depends on expected inflation:

\[ \ln \left( \frac{M}{P} \right)_t = a y_t - b i_t \]

where \( y_t \) is output and \( i_t \) is the interest rate.

Using the Fisher equation we may rewrite this as:

\[ \ln \left( \frac{M}{P} \right)_t = a y_t - b \left( r_t + \pi^e_{t+1} \right) \]

or:

\[ \ln M_t = \ln P_t + a y_t - b \left( r_t + \pi^e_{t+1} \right) \]
Differentiating this with respect to time, and assuming that output growth and real interest rate changes are relatively small compared to changes in nominal quantities in times of hyperinflations, we can get:

\[
\frac{d}{dt} \ln M_t = \frac{d}{dt} \ln P_t - b \frac{d}{dt} \ln \pi^e_{t+1} \\
\mu_t = \pi_t - b \dot{\pi}^e_{t+1}
\]

Or, rearranging:

\[
\pi_t = \mu_t + b \dot{\pi}^e_{t+1}
\]

If inflation is expected to increase in the future (i.e. \(\dot{\pi}^e_{t+1} > 0\)), then these expectations can cause inflation to increase to a level above \(\mu_t\).

Increases in inflation will cause individuals to increase their expectations of inflation, which in turn causes \(\pi_t\) to increase even further from.

The collapse in the demand for money is then partly caused by expectations of faster inflation, which are, ultimately, self-fulfilling.
Deficits and inflation

Let’s also take a quick glance at a famous result by Sargent and Wallace (1981) that monetary policy alone can be unsuccessful in controlling inflation when a government faces a binding debt ceiling (for whatever reason, to be discussed later).

Assumptions:

\[ P(G - T) + iB = \Delta M + \Delta B \quad \text{government budget constraint} \]
\[ M/P = kY \quad \text{QTM with constant velocity} \]
\[ Y, r \quad \text{exogenous (classical AS assumption)} \]
\[ b = \frac{B}{P} \leq \bar{b} \quad \text{real debt ceiling (say, investor fears)} \]

So the govt finances it’s spending (↑ \(G\)) by tax revenues (↑ \(T\)), new bond issuance (↑ \(B\)), money creation (↑ \(\mu \Rightarrow \pi\)) or cutting current debt repayment by defaulting on outstanding bonds (↓ \(iB\)).
The above implies that:

\[ \frac{\Delta M}{M} = \frac{\Delta P}{P} = \pi \]

**central bank controls**

\[ \frac{\Delta M}{M} \]

**government controls**

\[ G - T \]

The real government budget constraint (divided by \( P \)):

\[ G - T + i b = \frac{\Delta M}{M} \cdot \frac{M}{P} + \frac{\Delta B}{B} \cdot b \]

\[ \frac{\Delta B}{B} \approx \frac{\Delta b}{b} + \frac{\Delta P}{P} \]

\[ G - T + (r + \pi) b = \frac{\Delta M}{M} \cdot \frac{M}{P} + \left( \frac{\Delta b}{b} + \pi \right) b \]

\[ G - T + r b = \frac{\Delta M}{M} \cdot \frac{M}{P} + \Delta b \]
So we have:

\[ G - T + r b = \frac{\Delta M}{M} \cdot \frac{M}{P} + \Delta b \]

if an independent central bank sets money growth constant at a rate \( \mu \), then:

\[ G - T + r b = \mu k y + \Delta b \]

also note that:

\[ \mu = \frac{\Delta M}{M} = \frac{\Delta P}{P} = \pi \]
Phase I: Disinflation

\[ \frac{\Delta b}{b} = r + \frac{1}{b} \left( G - T - \mu k y \right) \]
Phase II: Monetisation of deficit

\[ \frac{\Delta b}{b} = r + \frac{1}{b} \left( G - T - \mu k y \right) \]
Some consistency between monetary policy and fiscal policy is required, at the very latest when public debt hits the ceiling, since:

- Either the government has to reduce its primary budget deficit or the central bank loses control of inflation:

\[
\mu = \pi = \frac{r \bar{b} + (G - T)}{k y}
\]

- If fiscal policy is too expansionary that then attempt of disinflation by the central bank will merely serve as \textit{inter-temporal substitution of inflation}.

- Long-run inflation may be beyond the exclusive control of the central bank.

- i.e. Sargent and Wallace (1981) propose that in the long run inflation is always and everywhere a fiscal phenomenon.
Rapid but non-explosive inflation

Cases of high and lasting inflation with explosion have also been observed. One explanations of such very high and stable inflation rates has been offered by Dornbusch (1992) in terms of a continuing but non-deteriorating government deficit.

Assume a government whose expenditure falls short of the money it can raise from taxation, and that it also has a poor credit rating and is thus unable to borrow from the capital markets.

The deficit thus can be financed only through money creation.

The deficit per unit time period is given by:

\[ \text{Deficit} = gPY \]

where \( P \) is the price level, \( Y \) is real output, and \( g \) is a “deficit rate” on nominal income.
This deficit is financed through increases in the stock of money:

$$\frac{dM}{dt} = gPY$$

Using the equation of exchange \((MV = PY)\) we can divide the above formula by \(MV\) on the left side and by the equal amount \(PY\) on the right side:

$$\frac{dM}{dt} \cdot \frac{1}{MV} = gPY \cdot \frac{1}{PY}$$

which produces:

$$\frac{1}{M} \frac{dM}{dt} = gV$$

rate of growth of the money supply
Assuming also that the rate of growth of the money supply equals the rate of growth of the price level (inflation), then we can rewrite the above as:

$$\pi = gV$$

For large values of $g$ and/or $V$ the inflation rate can be quite rapid, but in the simplest versions of this model there is no reason why it should accelerate.

A straightforward modification of the model is to allow the velocity of circulation to be increasing in the inflation rate, $V = V(\pi)$ (with $\frac{\partial V}{\partial \pi} > 0$), which may lead to a deteriorating trade-off between the deficit and inflation and to the possibility of instability.
If we are initially at point E’ and inflation increases, this will cause velocity to increase since $V = V(\pi)$.

Higher velocity will cause inflation to rise and the process continues with ever increasing inflation. Point E’ is therefore an unstable equilibrium.