EC3115 Monetary Economics
Lecture 13: Monetary policy under uncertainty

Anuar D. Ushbayev

International School of Economics
Kazakh-British Technical University
https://anuarushbayev.wordpress.com/teaching/ec3115-2015/

Tengri Partners | Merchant Banking & Private Equity
a.ushbayev@tengripartners.com – www.tengripartners.com

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Relevant reading

Must-read articles


Section 1

Introduction to policy under uncertainty
Up to now we have implicitly assumed that the central bank:

- knows the “true” structural model\(^1\) of the economy,
- knows the values of the model parameters with certainty,
- makes timely observes and accurate measures all relevant variables,
- knows sources and statistical properties of economic disturbances.

In practice, conducting monetary policy is **a lot more difficult**.

There is tremendous uncertainty about the true structure of the economy, the impact policy actions have on the economy, and even about the state of the economy itself.

\(^1\)Uncertainty over the structure of the economic model, or reduced-form model instability, which manifests itself through the Lucas critique – which we’ve already studied – is one form of model uncertainty that the monetary authorities must consider when deciding on policy.

We will now examine other forms of uncertainty that will determine which monetary variable the authorities should target and how aggressively the policy should be used in order to reach the policy goals.
“Unfortunately, actually to use such a strategy in practice, you have to use *forecasts*, knowing that they may be wrong. You have to base your thinking on some kind of a monetary theory, *even though that theory might be wrong*. And you have to attach numbers to the theory, knowing that *your numbers might be wrong*, and that *all you’ve got is a statistical average* anyway. We at the Fed have all these fallible tools, and no choice but to use them. It’s a tough world, but that’s the way it is.”


\(^1\)Vice Chairman of the Board of Governors of the Federal Reserve System (1994-1996).
“What can you do to try to guard against failure? There are two principles that monetary policy makers need to keep in mind. **First of all, be cautious. Don’t oversteer the ship.** If you yank the steering wheel really hard, a year later you may find yourself on the rocks.

**Second, you must have a long-run strategy in mind.** The Federal Open Market Committee meets eight times a year. You can’t be thinking only about what’s going to happen in the next six or seven weeks; that’s basically irrelevant to the monetary policy decision. You must think about a long-term strategy, execute the first step of that strategy, and then watch. You must be flexible and prepared to modify or even abandon your strategy if things look to be going wrong.”

“People often misunderstand and think that we can’t have a long-run strategy because of all these uncertainties and because the world is constantly changing. That is quite wrong. **You must have a long-run strategy, but you must be willing to modify it as new information becomes available.** Can this stitch-in-time strategy lead you into error anyway? You bet it can! But the other strategy – the Bunker Hill strategy – is sure to lead you into error. And that makes it, to me, a very easy choice.”


This advice for caution in monetary policy-making translates into a smooth adjustment of interest rates.

The reasons for smooth adjustment of interest rates can be due to:

- data uncertainty (additive),
- parameter uncertainty (multiplicative).
Section 2

Poole's model of (data) additive uncertainty
Economic agents (policy makers, financial agents, firms, households) possess information and form their forecast in real time. As it turns out, most macroeconomic data is subject to continuous revisions. We can define data revisions in the following way.

\[ X^f_t = X^p_t + r^f_t \]

where:
- \( X^f_t \) – is the final or true value of the same variable,
- \( X^p_t \) – is the statistical agency’s initial announcement of a variable that was realised at time \( t \),
- \( r^f_t \) – is the final revision which may potentially be never observed.

The data revisions, \( r^f_t \), can be due to:
- short run revisions based on additional source data or
- benchmark revisions based on structural changes or updating base year.
If those above revisions are done because of new data arrivals that are not forecastable at the time of the forecast (news), we can refer to “well behaved” revisions since there is nothing economic agents can do about these.

If, however, future data revisions are forecastable at the time of the forecast these are not well behaved revisions (noise).

By not making use of the forecastability of future revisions, economic agents would violate one of the main assumption of modern macroeconomics, that is rationality.

Well behaved revisions have three properties:

- Revisions should have a mean zero. i.e. initial announcement of the statistical agency should be an unbiased estimate of the final value of the data,
- Final revision should be unpredictable given the information set at the time of the initial announcement, and
- The variance of the final revision should be small compared to the variance of the final value of the data.
Aruoba (2008) shows vast empirical evidence that none of above-mentioned properties of well behaved revisions are full-filled in the US data.

- The measurement problem is more important for output series than it is for inflation or employment/unemployment series.
- We will now look at a case where we assume that the structure of the economy is known with certainty, but the economy may be subject to additive shocks and this creates data uncertainty.
Poole (1970) analysed the implication of adding such news shocks to both the IS and LM schedules.

Such shocks could come about from for instance changes in consumer tastes or government expenditures shocks on the IS side and stock market crashes and financial crises such as the collapse of LTCM in 1998 or a change in the central bank behaviour on the LM side.

Poole’s model assumes that the parameters and structure of the model are known with certainty, which is an unrealistic assumption, but allows the IS and LM schedules to be subject to zero-mean random errors, again the so called news shocks.
Define IS and LM schedules as:

\[ Y = a - bR + \epsilon \]
\[ M = c - dR + eY + \eta \]

where \( \epsilon \) and \( \eta \) are zero-mean additive IS and LM shocks whose variances are \( \sigma^2_\epsilon \) and \( \sigma^2_\eta \), respectively, and for simplicity we ignore the price level in the LM curve.

Alternatively, assume the monetary authorities can control real money balances, denoted by \( M \). The authorities can either set the money supply, \( M \), or the interest rate, \( R \), but not both.

With a downward-sloping money demand schedule, either \( R \) or \( M \) can be set and the other variable will have to change to allow the markets to clear.
If the authorities set the interest rate, then from the IS schedule, the expected value of output, $Y$, given $R$, denoted $\mathbb{E} [Y | R]$ will be:

$$\mathbb{E} [Y | R] = a - bR$$

Assume that the goal of the policy makers is to minimise the variance of output. From the above we deduce that:

$$Y - \mathbb{E} [Y | R] = \varepsilon$$

and so the variance of output given that the authorities set the interest rate will be

$$\text{Var} [Y | R] = \mathbb{E} \left[ (Y - \mathbb{E} [Y | R])^2 \right] = \mathbb{E} [\varepsilon^2] = \sigma^2_{\varepsilon}$$

(Alternatively, the monetary authorities could set the money supply, $M$. In order to calculate the variance of output in this scenario, we need to calculate $Y$ as a function of $M$ only.)
From the LM schedule, we can calculate $R$ as a function of $M$, $Y$ and $\eta$ and then substitute this into the IS curve.

\[
M = c - dR + eY + \eta
\]

\[
\Rightarrow R = \frac{M - c - eY - \eta}{d}
\]

Substituting this into the IS curve and solving for $Y$ gives:

\[
Y = a - b \left( -\frac{M - c - eY - \eta}{d} \right) + \epsilon
\]

\[
Y + \frac{beY}{d} = a + b \left( \frac{M - c - \eta}{d} \right) + \epsilon
\]

\[
Y (d + be) = ad + b \left( M - c - \eta \right) + d\epsilon
\]

\[
Y = \frac{ad + b \left( M - c - \eta \right) + d\epsilon}{d + be}
\]

\[
Y = \frac{ad - bc}{d + be} + \frac{b}{d + be} + \frac{d\epsilon - b\eta}{d + be}
\]
We can now of course also find $\mathbb{E}[Y|M]$ and $(Y - \mathbb{E}[Y|M])$:

$$\mathbb{E}[Y|M] = \frac{ad - bc}{d + be} + \frac{b}{d + be}$$

$$Y - \mathbb{E}[Y|M] = \frac{1}{d + be} (d\epsilon - b\eta)$$

and the variance of output given that the authorities could directly control the money stock in this model:

$$\text{Var}[Y|M] = \mathbb{E}[(Y - \mathbb{E}[Y|M])^2]$$

$$= \mathbb{E}
\left[
\left(\frac{1}{d + be} (d\epsilon - b\eta)\right)^2
\right]$$

$$= \left(\frac{1}{d + be}\right)^2 \left(d^2 \mathbb{E}[\epsilon^2] - db \mathbb{E}[\epsilon] \mathbb{E}[\eta] + b^2 \mathbb{E}[\eta^2]\right)$$

$$= \left(\frac{1}{d + be}\right)^2 \left(d^2 \sigma_\epsilon^2 + b^2 \sigma_\eta^2\right)$$
We can now examine which policy instrument, when set by the authorities, will result in a lower output variance.

\[
\begin{align*}
\text{(Only LM shocks, } \sigma_{\epsilon}^2 = 0) & \quad \text{Var} \left[ Y \mid R \right] = 0 < \text{Var} \left[ Y \mid M \right] = \left( \frac{b}{d+be} \right)^2 \sigma_{\eta}^2 \\
\text{(Only IS shocks, } \sigma_{\eta}^2 = 0) & \quad \text{Var} \left[ Y \mid R \right] = \sigma_{\epsilon}^2 > \text{Var} \left[ Y \mid M \right] = \left( \frac{d}{d+be} \right)^2 \sigma_{\epsilon}^2
\end{align*}
\]

Consider the case where there are no IS shocks, i.e. \( \sigma_{\epsilon}^2 = 0 \):

- Output variance under both interest rate and money targeting regimes is given in the top line of the table above.
- It is clear that setting the interest rate and allowing money to change to clear the market is the optimal strategy.

However, if there are no LM shocks, i.e. \( \sigma_{\eta}^2 = 0 \), then:

- Output variance is smaller under a policy of fixed money supply, as shown see the bottom line of the table.
Therefore, in this model:

- If an economy is prone to IS shocks, the authorities should keep the money supply constant.

- If the economy is prone to money market, LM, shocks, the interest rate should be the instrument of choice.

This can also be seen from the graphs below:
The left panel shows the case with IS shocks only.

The output variation when the interest rate is fixed at $R^*$ is shown by $|_R$, in which case the money supply has to change to clear the money markets causing the LM curve to shift so that equilibrium is at points A or B.

If the money supply was kept fixed then output deviations are shown by $|_M$.

Therefore, with IS shocks, in order to keep output variance at a minimum, it is best to keep the money supply fixed.
The right panel shows the case with LM shocks only.

By keeping the interest rate fixed after LM shocks, equilibrium will be unchanged at point E and output variance will therefore be zero.

By keeping the money supply fixed, however, LM shocks will cause equilibrium to move between points A and B and output variance will be positive, equal to $\Delta Y|_M$.

So, the main source of economic shocks (goods or money markets) will determine which monetary instrument the authorities should target.
Section 3

Brainard’s model of (parameter) multiplicative uncertainty
Whereas Poole considered the case where shocks were additive in nature, Brainard (1967) examined the case where the values of the parameters in the model were not known with certainty.

This is, arguably, more realistic since any model must be estimated from data.

An estimated model will not only give point estimates of the parameters but will also give standard errors since there will always be measurement error, model mis-specification and other problems that cause us not to know the exact structure of the economic model.
Suppose output, \( y \), depends on a policy mix, \( X \), a vector containing fiscal and monetary instruments that the government can control.

The relationship between \( y \) and \( X \) is given by:

\[
y = gX
\]

For simplicity, assume \( X \) is a single policy instrument so that \( g \) is a scalar parameter estimate with mean \( \hat{g} \) and variance \( \sigma_g^2 \).

The aim of the authorities is to minimize the variance of \( y \) around some target level, \( y^* \), full employment level of output for example, subject to the constraint:

\[
\min_X \mathbb{E} \left[ (y - y^*)^2 \right] \quad \text{s.t.} \quad y = gX
\]

We can deduce from above that:

\[
\hat{y} = \mathbb{E} \left[ y \right] = \mathbb{E} \left[ gX \right] = \hat{g}X
\]

\(^2\)You could alternatively formulate this for any choice of other target variable and policy instrument, e.g. \( \pi = gi \), where \( \pi \) is inflation, and \( i \) is the policy interest rate.
The above problem can then be rewritten as:

\[
\min_X \mathbb{E} \left[ (y - y^*)^2 \right] \quad \text{s.t.} \quad y = gX
\]
\[
\Rightarrow \min_X \mathbb{E} \left[ ((y - \hat{y}) - (y^* - \hat{y}))^2 \right]
\]
\[
\Rightarrow \min_X \mathbb{E} \left[ ((g - \hat{g})X - (y^* - \hat{g}X))^2 \right]
\]
\[
\Rightarrow \min_X \mathbb{E} \left[ (g - \hat{g})^2 X^2 - 2(g - \hat{g})X (y^* - \hat{g}X) + (y^* - \hat{g}X)^2 \right]
\]

Noting that \( \mathbb{E} \left[ (g - \hat{g})^2 \right] = \sigma_g^2 \) and \( \mathbb{E} \left[ g - \hat{g} \right] = \hat{g} - \hat{g} = 0 \), the problem becomes:

\[
\min_X \left( X^2 \sigma_g^2 + (y^* - \hat{g}X)^2 \right)
\]
Differentiating \(\left( X^2 \sigma_g^2 + (y^* - \hat{g}X)^2 \right) \) with respect to \(X\), the choice variable, and setting equal to zero:

\[
\frac{\partial}{\partial X} \left( X^2 \sigma_g^2 + (y^* - \hat{g}X)^2 \right) = 2X \sigma_g^2 - 2\hat{g} (y^* - \hat{g}X) = 0
\]

Solving for \(X\), this gives:

\[
2X \sigma_g^2 - 2\hat{g} (y^* - \hat{g}X) = 0
\]

\[
2X \left( \sigma_g^2 + \hat{g}^2 \right) = 2\hat{g} y^*
\]

\[
X = \frac{\hat{g} y^*}{\sigma_g^2 + \hat{g}^2}
\]

which implies:

\[
\hat{y} = \hat{g}X = \left( \frac{\hat{g}^2}{\sigma_g^2 + \hat{g}^2} \right) y^* < y^*
\]
“Brainard uncertainty principle”

- Policy thus faces a trade-off between reaching the target output and minimizing its variability.

- The implication of the model is that because of the presence of uncertainty \( \sigma_g^2 > 0 \), the authorities will never push aggressively enough to make average output equal to the target level, \( y^* \).

- To do so will simply cause output variation to increase to an intolerable level. The policy maker would rather have a stable level of output below the full employment level than very volatile output whose average was \( y^* \).

- The Brainard principle states that policy should exhibit caution/conservatism in the face of uncertainty.

- There are exceptions to this principle in the literature and one occurs when the uncertainty involves the degree of persistence of inflation. In such models, an aggressive response rather than a conservative response is optimal.
"Brainard uncertainty principle" illustrated

- $y = gX$ and $\hat{y} = \hat{g}X$ imply that $\sigma^2_y = \sigma^2_g X^2$, which means $X = \frac{\sigma_y}{\sigma_g}$.

- Substituting this into $y = gX = g \frac{\sigma_y}{\sigma_g}$ produces the linear policy constraint on the above graph.
"Brainard uncertainty principle" illustrated

- The authorities try to reach the indifference curve closest to $y^*$, the target level, subject to the policy constraint and as can be seen in the above graph.

- The presence of uncertainty, $\sigma_g^2$, causes the authorities to opt for a less aggressive policy stance, causing equilibrium output to be below $y^*$. 
Section 4

Parameter uncertainty in a New Keynesian model
In this section we will apply Brainard’s ideas to a simple model in the New Keynesian spirit.

We will show that if there is no parameter uncertainty, and the economy is subject to additive shocks with zero mean and constant variance, the policy maker should behave in a certainty equivalent manner, that is *as if* the economic shocks did not occur.

If, however, the economy is subject to structural changes as exemplified in parameter uncertainty, then Brainard conservatism applies.
Data (additive) uncertainty

- Suppose that the economy is characterised by:

\[
\begin{aligned}
\pi_t &= y_t + a\pi_{t-1} \\
y_t &= -bi_t + \epsilon_t
\end{aligned}
\]

where:

- $\pi_t$ is inflation,
- $y$ is the business cycle component of real output (or income),
- $i$ is the short term rate that the policy maker can control,
- $\epsilon$ is the stochastic demand shocks hitting the economy with zero mean and constant variance, i.e. $\epsilon \sim (0, \sigma^2_{\epsilon})$, known to the central bank, and
- $a$ and $b$ are constants.
Combining the above gives an expression for current inflation:

$$\pi_t = a \pi_{t-1} - bi_t + \epsilon_t$$

as a function of past inflation, the policy rate and the shocks hitting the economy.

The central bank cares about inflation stabilisation, with the following simple quadratic loss function:

$$L = (\pi_t - \pi^*)^2$$

where $\pi^*$ is target inflation.

In other words, whenever the current inflation deviates form the target inflation, the policy maker has an incentive to bring back the inflation to its target level by manipulating the policy rate, $i$. 
The only source of uncertainty is the presence of stochastic shocks. Given that the policy maker knows the nature of the shocks is \( \varepsilon \sim (0, \sigma^2_\varepsilon) \), it will form an expectation about these.

Substitute the perceived structure of the economy into the objective function of the central bank:

\[
L^e = \mathbb{E}_t \left[ (a\pi_{t-1} - bi_t + \varepsilon_t - \pi^*)^2 \right]
\]

\[
= a^2\pi^2_{t-1} + b^2i^2_t + \mathbb{E}_t \left[ \varepsilon^2_t \right] + \pi^*^2 - 2a\pi_{t-1}bi_t + 2a\pi_{t-1} \mathbb{E}_t \left[ \varepsilon_t \right]
\]

\[
= \sigma^2_\varepsilon + \pi^*^2 - 2a\pi_{t-1} \pi^* - 2bi_t \mathbb{E}_t \left[ \varepsilon_t \right] + 2bi_t \pi^* - 2 \mathbb{E}_t \left[ \varepsilon_t \right] \pi^*
\]

The job of the central bank now is to minimize this expected loss function with respect to the policy variable, the short term interest rate \( i_t \).
Differentiating with respect to the interest rate and setting to zero gives:

\[
\frac{\partial L^e}{\partial i_t} = 2b^2 i_t + 2b \pi^* - 2a \pi_{t-1} \ b = 0
\]

which is solved by:

\[
2b^2 i_t = -2b (\pi^* - a \pi_{t-1})
\]

\[
i_t = \frac{a \pi_{t-1} - \pi^*}{b}
\]

It is important to notice that the policy rate set by the policy maker is the same as the one that it would set if there were no shocks hitting the economy, since \( \sigma^2 \epsilon \) does not figure in the expression for \( i_t \).

This is the certainty equivalence result. The above means that additive shocks do not affect the way monetary policy is conducted. The best the policy maker can do is simply to ignore them.
Parameter (multiplicative) uncertainty

Now suppose we complicate the above model slightly by allowing parameter $b$ in the equation for real output to vary over time such that we now have:

\[
\begin{align*}
\pi_t &= y_t + a\pi_{t-1} \\
y_t &= -b_t i_t + \epsilon_t
\end{align*}
\]

where:

- $\epsilon$ is the stochastic demand shocks with $\epsilon \sim \left(0, \sigma^2_\epsilon\right)$, as before, but $b_t$ is no longer a constant, and instead is distributed as $b_t \sim \left(\hat{b}, \sigma^2_b\right)$, where once again we assume that the first two moments of $b$'s distribution are known to the central bank.

- Combining the above similarly gives an expression for current inflation:

\[\pi_t = a\pi_{t-1} - b_t i_t + \epsilon_t\]
The problem of the central bank now becomes:

\[ L^e = \mathbb{E}_t \left[ (a\pi_{t-1} - b_t i_t + \epsilon_t - \pi^*)^2 \right] \]

\[ = a^2 \pi^2_{t-1} + \mathbb{E}_t \left[ b_t^2 \right] i_t^2 + \mathbb{E}_t \left[ \epsilon_t^2 \right] + \pi^{*2} - 2a\pi_{t-1} \mathbb{E}_t \left[ b_t \right] i_t + 2a\pi_{t-1} \mathbb{E}_t \left[ \epsilon_t \right] \]

\[ = \sigma^2_\epsilon + \hat{b} \]

\[ = 0 \]

\[ -2a\pi_{t-1} \pi^* - 2 \mathbb{E}_t \left[ b_t \right] i_t \mathbb{E}_t \left[ \epsilon_t \right] + 2 \mathbb{E}_t \left[ b_t \right] i_t \pi^* - 2 \mathbb{E}_t \left[ \epsilon_t \right] \pi^* \]

\[ = 0 \]

\[ = a^2 \pi^2_{t-1} + \mathbb{E}_t \left[ b_t^2 \right] i_t^2 + \sigma^2_\epsilon + \pi^{*2} - 2a\pi_{t-1} \hat{b} i_t - 2a\pi_{t-1} \pi^* + 2\hat{b} i_t \pi^* \]

Using the definition of variance, we can replace the \( \mathbb{E}_t \left[ b_t^2 \right] \) term above as:

\[ \mathbb{E}_t \left[ b_t^2 \right] = \sigma^2_b + \left( \mathbb{E}_t \left[ b_t \right] \right)^2 = \sigma^2_b + \hat{b}^2 \]
The above expression for \( L^e \) then becomes:

\[
L^e = a^2 \pi^2_{t-1} + \left( \sigma^2_b + \hat{b}^2 \right) i_t^2 + \sigma^2_\varepsilon + \pi^* - 2a \pi_{t-1} \hat{b}i_t - 2a \pi_{t-1} \pi^* + 2\hat{b}i_t \pi^*
\]

Differentiating with respect to the interest rate and setting to zero gives:

\[
\frac{\partial L^e}{\partial i_t} = 2 \left( \sigma^2_b + \hat{b}^2 \right) i_t - 2a \pi_{t-1} \hat{b} + 2\hat{b} \pi^* = 0
\]

which is solved by:

\[
2 \left( \sigma^2_b + \hat{b}^2 \right) i_t = -2\hat{b} \left( \pi^* - a \pi_{t-1} \right)
\]

\[
i_t = \frac{\hat{b} \left( a \pi_{t-1} - \pi^* \right)}{\sigma^2_b + \hat{b}^2}
\]

\[
i_t = \frac{a \pi_{t-1} - \pi^*}{\hat{b} + \left( \sigma^2_b / \hat{b} \right)}
\]
Compare the cases without and with the uncertainty in parameter $b$:

$$i_t = \frac{a\pi_{t-1} - \pi^*}{b}$$

$$i_t = \frac{a\pi_{t-1} - \pi^*}{\hat{b} + \left(\frac{\sigma_b^2}{\hat{b}}\right)}$$

You should recognise the expression $\left(\frac{\sigma_b^2}{\hat{b}}\right)$ from your statistics classes as the coefficient of variation (CV), measuring the dispersion of $b$’s probability distribution.

It represents the trade-off between returning inflation to target, and the increasing uncertainty about inflation depends on the variance of parameter $b$ relative to its mean.

A large CV means that for a small reduction in the inflation bias the central bank induces a large variance into future inflation.

Once parameter uncertainty is taken into account inflation variance depends on the interest rate reactions.

The central bank’s decisions affect uncertainty of future inflation. Hence, a gradual policy reaction is preferable to an aggressive one.
Suppose that there is a negative relationship between real output (horizontal axis) and policy rates (vertical axis), which characterises the IS curve.

Above graph shows the case of no uncertainty.

As is clear whenever the actual output deviates from desired output, the policy maker can adjust policy rates to reach the desired level.
Graphical exposition (data uncertainty)

- Above graph shows the case with additive uncertainty (only news shocks are considered here).
- Here, the policy maker is not sure about the quality of the data it receives (as shown by parallel dashed lines).
- Nevertheless, it acts as if it knows the data with certainty, as there is nothing it can do about the shocks (certainty equivalence).
Graphical exposition (parameter uncertainty)

- Above graph shows the case with parameter uncertainty.
- Since, the nature of the relationship between output and policy rate is time-varying, the slope is changing over time.
- If the policy makers acts with aggression, that is, trying to reach the desired output by increasing the policy rate a lot, the economy may find itself at an equilibrium that is even less desirable than before.
What can the policy maker do?

- Sack (2000) demonstrates that a possible alternative to an aggressive policy that can potentially result in an undesired negative outcome is that of *gradualism* in interest rate-setting:
  - The policy maker moves in small steps in the same direction for consecutive periods (interest rate smoothing):
    - First, the central bank adjusts the policy rates in the right direction.
    - Later, when the (relatively benign) state of the economy is revealed, it moves again in the same direction.
  - The policy maker thereby exploits arrival of new information and can reduce the size of potential errors it can commit.
Interest rate smoothing

**Single Policy Step**

**Two Policy Steps**

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ISE – KBTU

A.D. Ushbayev (2016)
Is Brainard uncertainty empirically relevant?

- Sack (2000) has studied the Fed’s behaviour and found that it has historically adjusted policy rates in a smooth fashion, in large part due to parameter uncertainty, as well as the uncertainty about the state of the economy.

- This deliberate pace of movements in policy interest rates has been considered as evidence of an interest-rate smoothing incentive on the part of central banks.

- Under this interpretation, the central bank is reluctant to change the policy rate aggressively, choosing instead to move the interest rate tentatively towards its new level.

- Sack (2000) models the structure of the economy using a five dimensional vector autoregression in the industrial production growth rate, the unemployment rate, consumer price inflation, commodity price inflation (to control for price puzzles) and the federal funds rate.
Structure is imposed by a recursive Choleski factorization in which the federal funds rate is ordered last.

Assuming the model is correctly identified, the first four equations in the model describe the structural form of the economy whilst the last equation is the estimated policy function of the Fed\(^3\).

The following objective function is assumed for the Fed:

\[
L = -\frac{1}{2} \mathbb{E}_t \left[ \sum_{i=1}^{\infty} \beta^i \left( (\pi_{t+i} - \pi^*)^2 + \lambda_u (u_{t+i} - u^*)^2 + \lambda_y (y_{t+i} - y^*)^2 \right) \right]
\]

where the weights \(1, \lambda_u\) and \(\lambda_y\) determine the relative importance of the deviation of inflation, unemployment and production growth from their respective targets.

\(^3\)Because this is not a true structural model based on microfoundations it may be subjected to the Lucas critique problems.
Notice that the objective function does *not* contain any explicit reason to smooth interest rates, since the purpose was to investigate whether gradual funds rate movements can be explained without simply assuming that the Fed prefers to act gradually.

After estimating the model with OLS, Sack (2000) calculates a “certainty equivalent” policy rule by assuming that the point estimates from the VAR are the true values known with certainty.

This rule will be linear in the past values of the variables in the system since it is a standard linear-quadratic control problem.

The coefficients depend on preferences $\lambda_u$ and $\lambda_y$, and the point estimates of the VAR coefficients.
This rule is then compared to a “Brainard” policy rule which takes into account that the point estimates of the VAR are uncertain.

The standard OLS errors of the parameter estimates are used as an indicator of the uncertainty surrounding each parameter.

The new optimal rule is still a linear function of past values of variables in the system but now the coefficients of the rule depend on both the point estimates of the VAR and the variance-covariance matrix of the coefficient estimates.

Sack (2000) then calculates the impulse response functions implied by the optimal rules with and without allowance for parameter uncertainty and compares these to those estimated purely from the data.
He finds, as the graph clearly shows, that the optimal policy rule taking parameter uncertainty into account (rather than disregarding) tracks the federal funds rate better, suggesting that caution induced by Brainard uncertainty is quantitatively important.

Both of the optimal policy rules exhibit considerable persistence in interest rates which is a suggestion for gradualism.
Section 5

Model uncertainty
The analysis above still assumes that the policy-makers know precisely how uncertain they are – to devise optimal policy rules, central banks must know

- the variance of the uncertain structural parameters and additive shocks,
  and

- the nature of measurement errors.

A more realistic assumption may be that uncertainty is more pervasive than this.

In particular, and fundamentally, policy-makers are uncertain about the basic form of the “true” model of the economy. This would be the case, for example, if it was unclear which variables to omit or include in the model.

Theorists have considered how policy should be set, given such “model uncertainty”.
Model averaging and robustness

- One idea is that policy rules could be designed that perform well across a range of plausible fully specified models of the economy.

- Such “robust” policy rules would not, by definition, perform as well as an optimal rule designed for a particular model.

- But they would be designed to perform quite well both with this model and a range of similar models, whereas the optimal rule might perform poorly with other models.

- Sargent (1999) takes a simple macro-policy model similar to above and calculates a policy rule robust to small mis-specifications around this model.

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Unlike the Brainard conservatism result for parameter uncertainty, Sargent finds that the robust rule responds more aggressively to shocks than the certainty-equivalent optimal rule in the model.

The intuition for this result in Sargent (1999) is that, by pursuing a more aggressive policy, the central bank can prevent the economy from encountering situations where model mis-specification might be especially damaging.

For the simple model presented above, it is possible to show that a “Sargent robust rule” coincides with the certainty-equivalent optimal rule.